IRREGULAR MOTION OF PARTICLES IN A TURBULENCE CHAMBER

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An examination is made of the irregular motion of a solid particle in a turbulence chamber. Theoretical results obtained are compared with experimental data.

Many industrial processes involve the motion of solid particles in rotating flows of a continuous medium. Information on the character of motion of the particles and their velocities and paths is of definite interest for improving the production processes. We will examine the motion of a particle in a rotating gas flow bounded by a lateral surface and two end covers in a turbulence chamber (the tentative term for all equipment with a swirling flow of a continuous medium). With the presence of a central sink, the particle is acted upon by a radial resistance force which may compensate for the centrifugal force present, and the particle will move in an equilibrium orbit if the orbit radius r is less than the chamber radius R [1]. If r > R, the particle will intensively interact with the walls, executing more or less periodic jumps along the generatrix of the chamber (Fig. 1). Here, particle velocity decreases significantly as a result of the loss in the impacts with the wall, and the particle begins to lag behind the carrier flow. The interaction of the particle with the wall also gives the particle an angular velocity of rotation $\omega$ and thus creates a Magnus force transverse to the velocity. The possible role of the Magnus force in the generation of irregular particle motion and its consequent deceleration was examined in detail in [1], where it was shown that in a certain range of values of the criterion $\alpha = 5h\rho/\rho_1 d$ the particle is effectively slowed as a result of its interaction with the end walls of the chamber. In sufficiently high turbulence chambers, particles move in the plane of the chamber (Fig. 1) and interact mainly with its lateral surface. At first glance, it is not clear what causes the irregular particle motion because the particle does not move along the chamber generatrix at the velocity of the carrier flow but lags significantly behind it. In the present article, we propose a simple model which can be used to successfully explain and describe these phenomena concerning particle motion in a turbulence chamber.

The equation of particle motion, subject to a quadratic resistance law, has the form

$$\frac{d\mathbf{w}}{dt} = \lambda\mathbf{u} + k[\mathbf{u}\times\boldsymbol{\omega}],\ \mathbf{u} = \mathbf{V} - \mathbf{w},$$
$$\lambda = \frac{3\zeta\rho}{4d\rho_1}|u|,\ k = k_1\rho/\rho_1,\ |u| = [(V_r - w_r)^2 + (V_\varphi - w_\varphi)^2 + (V_z - w_z)^2]^{1/2}. \quad (1)$$

If the particle interacts with the lateral surface of the chamber, executing more or less regular jumps, then with each impact it loses some of its radial momentum (with a restitution coefficient n < 1). In the case of steady motion, the lost portion of radial velocity should be compensated for by the work done by the resistance force of the flow and the Magnus force. It is not hard to show that the work done by the Magnus force in the case of a central sink is negative, i.e., to maintain the irregular motion work must be completed by the resistance force. On the other hand, steady irregular particle motion can be maintained only when the angle of movement of the particle toward the wall is less than the angle of rebound (the angle between a normal to the wall and a tangent to the particle path at the site of its impact with the wall). In this case, the radial components of both the Magnus force and the resistance force are directed toward the center, and the sign of the curvature of the path coincides with the corresponding sign for the wall, i.e., the above-mentioned angle of movement toward the wall (angle of incidence) increases from impact to impact for n < 1, and the particle should roll over the cylindrical lateral surface at the end of the ends. In actual

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turbulence chambers, the side differs from a cylindrical surface by the presence of slits in the guide, which can serve to generate jumps solely as a result of the geometry of impact. It will be shown below that even a very slight difference between the lateral surface and a cylindrical surface gives rise to irregular particle motion and significantly slows the particle.

We will study the planar steady motion of a solid spherical particle in a turbulence chamber with a ribbed surface (Fig. 2). We will assume that the distance between adjacent ribs is the sought quantity, with specified values of the parameters of the carrier flow and particle, as well as for the restitution coefficient and the angle of rotation $\theta$ of the plane of the ribs relative to a tangent to an inscribed cylinder. As the carrier flow we will examine a gas ($\rho/\rho_1 \ll 1$) moving in accordance with the law for a solid $V_\varphi = \varkappa r$, $\varkappa = V_0/R$. The radial velocity of the gas will be ignored in comparison with the azimuthal velocity. Neither of these conditions alters the physical significance of the problem and can be changed without loss of simplicity and clarity in the solution. With these goals in mind, let us first omit consideration of the Magnus force. We will subsequently obtain a solution with allowance for this force.

The equations of motion of the particle in a Cartesian coordinate system have the form

$$\frac{dw_x}{dt} = \lambda (V_x - w_x),\ V_x = -\varkappa y,$$
$$\frac{dw_y}{dt} = \lambda (V_y - w_y),\ V_y = \varkappa x. \tag{2}$$

If we introduce the complex particle coordinate $z = x + iy$, then system (2) reduces to a single second-order differential equation (see [3]):

$$\ddot{z} + \lambda \dot{z} - i\varkappa\lambda z = 0, \tag{3}$$

whose solution is as follows:

$$z = A\exp(\mu_1 t) + B\exp(\mu_2 t),$$

where

$$\mu_{1,2} = -\frac{\lambda}{2}(1 \pm \sqrt{1+i\beta}),\quad \beta = \frac{4\varkappa}{\lambda}\left(\equiv \frac{16}{3\zeta}\frac{V_0}{|u|}\frac{\rho_1 d}{\rho R}\right).$$

Here we linearized the problem with respect to particle velocity, since in accordance with experiments $w_r, w_\varphi \ll V_0$, and it is assumed that $|u| = \mathrm{const}\ (\sim V_0)$. For sufficiently large particles, the parameter $\beta \gg 1$, and the roots of the characteristic equation are equal to

$$\mu_{1,2} = \pm\mu,\ \mu = \frac{\lambda}{2}\sqrt{\frac{\beta}{2}}(1+i).$$

We will examine the motion of a particle between two successive impacts corresponding to the moments of time $t = t_1$, $t_1 + T$, and we will introduce the time $\tau \in [0, T]$. The state of the particle after impact will be designated by the subscript "0," while its state at the moment before impact will be designated by the subscript "1," i.e., $z(0) = z_0$, $z(T) = z_1$. Let $z_0 = R$. At the moment of subsequent impact

$$z_1\bar{z}_1 = R^2. \tag{4}$$

We write the solution of Eq. (3):

$$z = \frac{w_0}{\mu}\,\mathrm{sh}\,\mu\tau + R\,\mathrm{ch}\,\mu\tau,$$
$$w = w_0\,\mathrm{ch}\,\mu\tau + \mu R\,\mathrm{sh}\,\mu\tau,\ w_0 = w_{0x} + iw_{0y}. \tag{5}$$

The condition of periodicity of the particle motion has the form

$$w'_{0\varphi} = w'_{1\varphi},$$
$$w'_{0r} = -nw'_{1r}, \tag{6}$$

where the prime denotes a system of coordinates in which particle velocity $w'_{1r}$ is normal to the wall at the moment of impact and $w'_{1\varphi}$ is the tangential velocity. The relationship between the coordinate systems is given by the equation

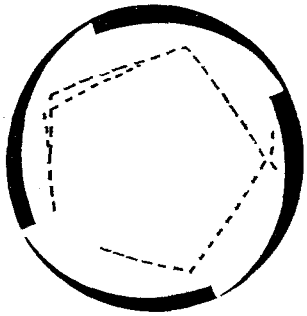

Fig. 1

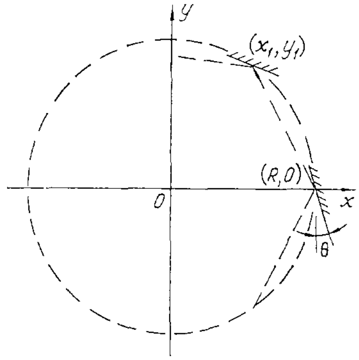


Fig. 2

Fig. 1. Path of steady-state motion of particle of sugar sugar with d = 3 mm in a turbulence chamber of radius R = 0.4 m and height h = 0.6 m (see [2]).

Fig. 2. System of coordinates and diagram of steady-state motion of particle.

$$w' = \exp(-i\theta)\, w_*,$$

where $w_*$ and $w'$ are complex velocities of the particle $(w_* = w_r + iw_\varphi)$ in a cylindrical coordinate system. The particle velocities in the cylindrical and Cartesian systems are connected by the equation

$$w_* = w\bar{z}/R,\ \bar{z} = x - iy. \tag{7}$$

Conditions (6) appear as follows in complex form

$$w_{0*}\exp(-i\theta) = -nw_{1*}\exp(-i\theta) + i(1+n)\,\mathrm{Im}\,(w_{1*}\exp(-i\theta)). \tag{8}$$

Changing over by means of Eq. (7) to the Cartesian coordinate system (Fig. 2), we use Eq. (4) and the form of the solution (5) to write condition (8) as follows:

$$\frac{\mu\exp(-i\theta)}{\mathrm{sh}\,\mu T}(R\,\mathrm{ch}\,\mu T - \bar{z}_1) = -w_0\exp(-i\theta)/n + \frac{i(1+n)}{n}\,\mathrm{Im}\left[\frac{\mu\exp(-i\theta)}{\mathrm{sh}\,\mu T}(R\,\mathrm{ch}\,\mu T - \bar{z}_1)\right]. \tag{9}$$

This relation and condition (4) give a closed system of equations for determining $w_0$ and T.

We will evaluate the modulus of the product $\mu T$. The period $T \sim R/N\hat{w}$, where N is the number of impacts per particle revolution about the axis of the turbulence chamber and $\hat{w}$ is the mean azimuthal velocity of the particle. Then, using the expression $|\mu| = (\lambda\varkappa)^{1/2}$, we will have $|\mu T| \simeq (V_0/N\hat{w})\beta^{-1/2} \ll 1$ for $\beta \gg 1$ (see above). The angle of inclination of the slits in the chamber is generally very small, so that we can use the smallness of $\mu T$ and $\theta$ to write Eq. (9) term by term in the form of a linear system of equations in $w_{0x}$ and $w_{0y}$:

$$\begin{gathered} w_{0x} - \theta(\gamma + w_{0y}) = (w_{0x} + \theta w_{0y})/n, \\ w_{0y} - \theta w_{0x} = (\gamma + w_{0y}) + \theta w_{0x}. \end{gathered} \tag{10}$$

Here $\gamma = \lambda\varkappa RT$. For $\theta = 0$, it follows from (10) that T = 0, $w_{0x} = 0$ $(n \neq 1)$, i.e., the particle rolls over the side of the chamber. The solution of system (10)

$$\begin{gathered} w_{0x} = -\gamma/(2\theta), \\ w_{0y} = \left(\frac{1-n}{1+n}\frac{1}{2\theta^2} - \frac{n}{1+n}\right)\gamma \end{gathered} \tag{11}$$

will be used to determine T from Eq. (4), which, by virtue of the fact that $z_1 = w_0 T + R$, has the form

$$T(w_{0x}^2 + w_{0y}^2) + 2Rw_{0x} = 0.$$

It follows from the last equation and (11) that

$$T = \frac{2}{(\lambda\varkappa)^{1/2}}[\theta/\nu(\theta,\ n)]^{1/2},\ \ \nu = 1 + \left(\frac{1-n}{1+n}\frac{1}{\theta} - \frac{2n\theta}{1+n}\right)^2. \tag{12}$$

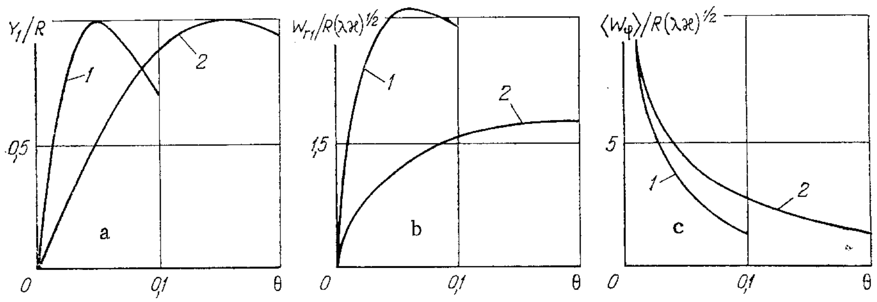


Fig. 3. Dimensionless diagrams showing coordinate of particle impact against the side of the chamber (a); radial (b) and azimuthal (c) velocities of particle in relation to the angle of rotation of the rib: 1) n = 0.9; 2) 0.7.

Finally, we have the following expressions for the coordinates of the impact of the particle against the wall at the moment of time T and the rebound velocity

$$x_1/R = 1 - 2/\nu, \quad y_1/R = 2(\nu - 1)^{1/2}/\nu,$$
$$w_{0x}/R(\lambda\varkappa)^{1/2} = -(\theta\nu)^{-1/2}, \quad w_{0y}/R(\lambda\varkappa)^{1/2} = [(\nu - 1)/(\theta\nu)]^{1/2}. \tag{13}$$

Given our assumptions, the coefficient $R\,(\lambda\varkappa)^{1/2} = V_0\left(\frac{3\zeta}{4}\,\frac{\rho R}{\rho_1 d}\right)^{1/2}$. The mean radial and azimuthal velocities are, respectively, equal to:

$$\langle w_r \rangle = \frac{1}{2RT}\int_0^T (w\bar{z} + \bar{w}z)\,d\tau = \frac{1}{2RT}\,z\bar{z}\Big|_0^T = 0,$$

$$\langle w_\varphi \rangle = \frac{1}{2iRT}\int_0^T (w\bar{z} - \bar{w}z)\,d\tau = R(\lambda\varkappa)^{1/2}F(\theta, n),$$

$$F(\theta, n) = [(\nu - 1)^{1/2} + \theta - (4\theta)/(3\nu)]/(\theta\nu)^{1/2}.$$

It follows from Eq. (11) that the angle of inclination of the slits should not exceed a certain value $\theta < [2n/(1 - n)]^{1/2}$. For $\theta \ll (1 - n)/(1 + n)$, the asymptotic expressions for the velocities are, respectively, equal to:

$$\langle w_\varphi \rangle / R(\lambda\varkappa)^{1/2} \to (1/\theta)^{1/2},$$
$$w_{1r}/R(\lambda\varkappa)^{1/2} \to \frac{1+n}{1-n}\,\theta^{1/2}.$$

Figure 3 offers a graphical illustration of the results obtained. Since the parameter $R(\lambda\varkappa)^{1/2}/V_0 \ll 1$, it is apparent that even with a large restitution coefficient, a small angle of inclination of the slits leads to very significant deceleration of the particle. With a certain value of the angle θ dependent on the restitution coefficient, the radial velocity associated with particle impact against the wall passes through an extremum and reaches its greatest value. The greater the restitution coefficient, the greater this extremum.

We now introduce the Magnus force into the discussion. If by analogy with the above we use a complex particle coordinate, then system (1) reduces to Eq. (3), in which we need to replace λ by $\lambda_1 = \lambda + ik\omega$. The roots of the characteristic equation are equal to:

$$\mu_{1,2} = -\frac{\lambda}{2}\left\{1 \pm \left[\left(1 + \frac{4\varkappa k\omega}{\lambda^2 + (k\omega)^2}\right)^2 + \left(\frac{4\lambda\varkappa}{\lambda^2 + (k\omega)^2}\right)^2\right]^{1/4} \exp(i\chi)\right\},$$

$$\operatorname{tg}\chi = \frac{4\lambda\varkappa}{\lambda^2 + 4\varkappa k\omega + (k\omega)^2}.$$

The angular velocity of the particle is determined by the equation

$$w_{0y}' = (\omega d)/2$$

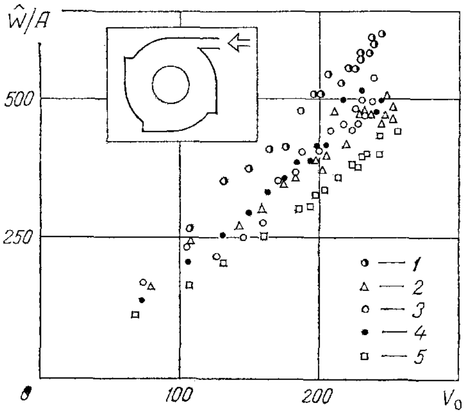


Fig. 4. Mean velocity $\hat{w}$ (m/sec) of glass and steel beads of different diameters in a turbulence chamber of radius R = 33 mm and height h = 12 mm in relation to the gas velocity in the slit $V_0$ (m/sec): 1) glass, n = 0.8, d = 3.1 mm; 2-4) steel, n = 0.6, d = 1.55; 2.5; 4.9 mm; $A = [(3\zeta/4)(\rho R/\rho_1 d)]^{1/2}$, $\zeta = 0.44$; 5) steel cubes, d = 3.5 mm.

in the case when impact occurs without slip, which will be assumed. The coefficient $k_1$ in the expression for the Magnus force is a quantity which is to some degree indeterminate, although several theoretical and empirical studies (see [1]) give $k_1 = 2$. We will show that allowing for the Magnus force does not lead to irregular motion of the particle when $\theta = 0$.

Let

$$\lambda/(k\omega) = \frac{3\zeta}{8k_1}\frac{|u|}{w_{0\varphi}} \ll 1, \text{ while } \frac{4\varkappa}{k\omega} = \frac{2V_0}{kw_{0\varphi}}\frac{\rho_1 d}{\rho R} \gg 1.$$

The first condition corresponds to the desired solution when at

$$\theta \to 0 \quad w_\varphi \to V_\varphi,\ w_r \to 0 \text{ and} |u| < w_{0\varphi}.$$

The second corresponds to the usual experimental situation. Then

$$\mu_{1,2} = \pm\mu,\ \mu = \left(\frac{\varkappa}{k\omega}\right)^{1/2}(\lambda/2 + ik\omega).$$

Using the procedure of expansion of condition (9) in the triviality $|\mu T| \sim \frac{V_0}{N\hat{w}}\sqrt{\frac{k\omega}{\varkappa}} \ll 1$ and $\theta \ll 1$, we obtain the system of equations:

$$(\eta_r - w_{0x}) + (w_{0y} + \eta_e)\,\theta = -(w_{0x} + \theta w_{0y})/n,$$
$$\eta_e = (\eta_r - 2w_{0x})\,\theta.$$

Here

$$\eta_r = R\frac{\varkappa}{k\omega}[(\lambda/2)^2 + (k\omega)^2]\,T \simeq \varkappa k\omega RT,\ \ \eta_e = \varkappa\lambda RT,$$

from which at $\theta = 0$ we have $T = 0$, $w_{0x} = 0$ at $n \neq 1$, i.e., the particle rolls over the lateral surface.

Thus, introduction of the Magnus force into the study does not qualitatively alter the physical mechanism of generation of irregular motion of the particle. It changes only the quantitative parameters of its motion. However, since there is no unanimous agreement regarding quantitative evaluation of the Magnus force and since this question requires independent study, in the present case we will limit ourselves to a qualitative conclusion on the mechanism of particle motion formulated above.

Figure 4 shows experimental data on the mean azimuthal velocity of particles in a turbulence chamber. Mean particle velocity $\hat{w}$ was evaluated as the rate of travel about the perimeter of the chamber during one revolution about its axis. The period of rotation was determined from the signal from a photomultiplier as the particle passed through a radial "light knife." In accordance with the theoretical model presented above:

$$\hat{w}/R\,(\lambda\varkappa)^{1/2} = \left[\arccos\left(\frac{\nu-2}{\nu}\right)\right]/(4\theta/\nu)^{1/2}\,. \qquad (14)$$

The mean particle velocity, according to (14), is a monotonic function of the restitution coefficient. At $n=1$ $\hat{w}/R(\lambda\varkappa)^{1/2} \simeq \pi/(2\theta^{1/2})$, while for $n \to 0$ $\hat{w}/R(\lambda\varkappa)^{1/2} \to \theta^{-1/2}$, i.e., $\hat{w}$ is fairly weakly dependent on n. In accordance with (14), glass beads, with a restitution coefficient greater than steel beads, moved at somewhat greater mean velocities (Fig. 4). It was difficult to establish quantitative agreement between the theory and the experiment, since we did not determine the angle $\theta$ for a specific experimental scheme (see the scheme in Fig. 4). However, if we compare Eq. (14) and the test data for the steel beads, then for a certain mean angle we obtain the value $\theta \sim 0.4$ (23°); this corresponds fully to the actual situation.

In conclusion, we should note that the results obtained depend little on the form of the particle. Points 5 in Fig. 4 correspond to steel cubes with an equivalent diameter $d \simeq 3.5$ mm. The moderate difference in the proportionally factor from the case of steel beads (spheres) is possibly connected with the somewhat different values of the resistance and restitution coefficients for the cubes. The geometry of the chamber is evidently the determining factor for the characteristics of particle motion, other conditions being equal as determined by the above-developed theory.

## NOTATION

R, h, radius and height of turbulence chamber; $\theta$, angle of rotation of plane of rib relative to a tangent to an inscribed cylinder; **V**, $\rho$, velocity of gas and its density; d, $\rho_1$, **w**, $\omega$, diameter of particle, its density, linear velocity, and angular velocity relative to the center of mass; **u**, relative velocity of particle and gas; $z = x + iy$, complex coordinate of particle; t, $\tau$, time; T, period of time between impacts; n, restitution coefficient; $\zeta$, coefficient of resistance of particle; $k_1$, numerical coefficient in the Magnus force; $\alpha$, $\beta$, dimensionless criteria.

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# CONVECTIVE HEAT EXCHANGE IN TURBULENT FLOW OF A GAS SUSPENSION WITHIN A CYLINDRICAL CHANNEL

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A convective heat-exchange model is presented for flow of a gaseous suspension in a tube, which considers the increase in heat capacity of the system and the effect of particles on the turbulent structure of the flow. Comparison of calculated results with experiment shows good agreement.

The intensity and efficiency of many heat-exchange processes in metallurgy, energy generation, and other branches of industry are determined by phenomena occurring in gas—solid particle type dispersed systems. Heat exchange was treated in [1] with consideration of the effect of particles on the turbulent structure of the carrier flow within the framework of Buevich's model [2], which consists of breakoff of the shortwave portion of the turbulent energy spectrum. The calculation results of [1] agree well with experiment in the low particle concentration range $\mu \leqslant 6$. It is of interest to consider the case of higher particle concentrations.

We will write the energy equation for the turbulent flow of a gas suspension in a tube just as in [1], but without consideration of radiation

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